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## Heat loss through cylindrical and spherical building partitions

Natalia Brycht<sup>1</sup> (*orcid id: 0000-0002-7372-7492*)

<sup>1</sup> Czestochowa University of Technology, Faculty of Civil Engineering

**Abstract:** Heat transfer through curvilinear partitions differs, in mathematical terms, from transfer through flat ones. However, in practical calculations, an approximation is commonly used by estimating heat loss by analogy to flat partitions. There are no studies in the literature related to an exact quantitative analysis of heat loss through curvilinear partitions. The aim of the article is a comparative analysis of the computational heat loss determined for layered cylindrical and spherical partitions determined by the exact and the approximate methods. The percentage discrepancy of results was determined. Relevant computational examples were analysed. It was found that in most cases the above approximation does not lead to significant inaccuracies if the basis for calculating the heat transferring surface area is the mean radius of curvature of the partition.

**Keywords:** heat loss in buildings, thermal transmittance, cylindrical and spherical building partitions

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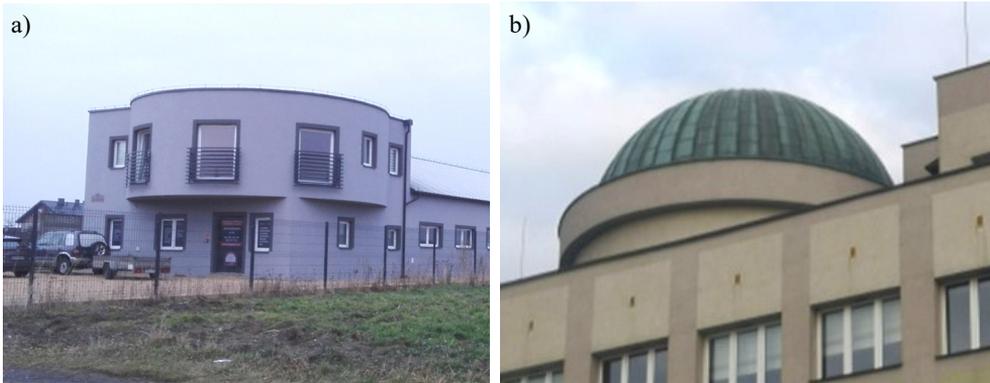
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### Introduction

Over the past several decades, there have been significant changes in urban and rural building features. Simple buildings used for years (Chen, 2011; Grycel, 2007; Rutkowska & Beba, 2006) are increasingly giving way to objects with more complex shapes. External building partitions can be designed in a curvilinear form, which is evident in contemporary architecture (Maciejko, 2016; Makachia, 2011; Szołomicki & Golasz-Szołomicka, 2017). Examples of layered building partitions in the form of a cylindrical wall and a dome are presented in Figure 1.

The shape of a building's form has an impact on heat loss through external partitions. For example, in the article (Respondek, 2016) it was shown that buildings with cylindrical, hemispherical or elliptical partitions lose less heat at a particular heated cubature than buildings with straight walls.



**Fig. 1.** Exemplary external curvilinear partitions: a) cylindrical wall, b) spherical flat roof  
(author's study)

Curvilinear forms of building partitions also influences the method of calculating heat loss through these partitions. In the case of layered flat partitions, heat loss is determined by thermal transmittance  $U$  [ $\text{W}/\text{m}^2\cdot\text{K}$ ], which can be defined as the density of heat-flow rate for a unit temperature difference on both sides of the partition. In the case of curvilinear partitions, it is not possible to operate with such a defined value, because even in the thermal steady state, the density of heat-flow rate changes, depending on the location of the investigated point in a cross-section. The temperature distribution in the thickness of the partition is also different. In flat partitions, the temperature changes linearly in every layer. In curvilinear partitions, the temperature changes non-linearly, and the theoretical model for determining heat loss is presented in many publications, including (Pogorzelski, 1976; Wymiana ciepła, 2019). Analyses of numerical models of this phenomenon are known, e.g. (Nagendra et al., 2014; Öztop, 2005), however, it should be noted that these analyses are mainly associated with the flow of fluids through conduits.

In practical calculations of building partitions, heat loss through curvilinear partitions is mostly calculated by an analogy with flat partitions, and exact mathematical formulas are rarely used in the construction industry. Calculation templates as well as spreadsheets in computer programs are made assuming a flat partition.

The aim of the article is a comparative analysis of the computational heat loss determined for layered building partitions with a regular radius of curvature (i.e. cylindrical and spherical), and determined by an approximate method by an analogy to flat partitions. The percentage discrepancy between the values estimated by exact and approximate methods was determined.

## 1. Methodology of research

As already mentioned, in the literature there are solutions used to calculate heat loss for layered curvilinear partitions. They are used in practice to calculate heat loss through pipelines (cylindrical partitions) and tanks (spherical partitions). It should

be added that exact mathematical solutions based on the solution of differential equations can be obtained for partitions with a constant radius of curvature. These formulas for layered cylindrical and spherical partitions are presented in Table 1. By analogy, these formulas can be adapted for heat loss calculations through building partitions of such shapes.

The formulas for calculating thermal transmittance and heat loss through flat, cylindrical and spherical partitions are summarized in Table 1. In the literature, these formulas were derived on the basis of a solution of differential equations.

**Table 1.** Thermal transmittance and heat loss for flat, cylindrical and spherical partitions, (PN-EN ISO 6946:2017-10; Pogorzelski, 1976; Wymiana ciepła, 2019)

Type of partition	Thermal transmittance	Heat loss
Flat partition	$U_p = \frac{1}{\frac{1}{\alpha_{si}} + \sum_j \frac{d_j}{\lambda_j} + \frac{1}{\alpha_{se}}}$	$Q_p = U_p \cdot F \cdot (t_i - t_e)$
Cylindrical partition	$k_L = \frac{1}{\frac{1}{\alpha_{si} \cdot D_0} + \sum_{j=1}^n \frac{1}{2\lambda_j} \ln \frac{D_j}{D_{j-1}} + \frac{1}{\alpha_{se} \cdot D_n}}$	$Q_L = k_L \cdot (t_i - t_e) \cdot \pi$
Spherical partition	$k_{sph} = \frac{1}{\frac{1}{\alpha_{si} \cdot D_0^2} + \sum_{j=1}^n \frac{1}{2\lambda_j} \left( \frac{1}{D_{j-1}} - \frac{1}{D_j} \right) + \frac{1}{\alpha_{se} \cdot D_n^2}}$	$Q_{sph} = k_{sph} \cdot (t_i - t_e) \cdot \pi$

The following symbols were used in Table 1:

$U_p$  – the thermal transmittance for a flat layered partition [ $W/(m^2 \cdot K)$ ];

$Q_p$  – the heat loss through a flat partition [ $W$ ];

$k_L$  – the substitute thermal transmittance for a layered cylindrical partition with a unit length (or height) [ $W/(m \cdot K)$ ];

$Q_L$  – the heat loss through a layered cylindrical partition with a unit length (or height), [ $W/m$ ];

$k_{sph}$  – the substitute thermal transmittance for a layered spherical partition [ $W/(m^2 \cdot K)$ ];

$Q_{sph}$  – the heat loss through a layered spherical partition [ $W$ ];

$n$  – the number of layers in a partition;

$j = 1 \dots n$  – the index denoting the next layer of the partition (from the inside);

$d_j$  – the thickness of layer with index  $j$  [ $m$ ];

$\lambda_j$  – the thermal conductivity of layer with index  $j$  [ $W/(m \cdot K)$ ];

$D_0$  – the internal diameter of a cylindrical or spherical partition [ $m$ ];

$D_j$  – the external diameter of layer with index  $j$  [ $m$ ];

$D_n$  – the external diameter of a cylindrical or spherical partition [ $m$ ];

$F$  – the surface area of the partition [ $m^2$ ];

$t_i, t_e$  – the internal and external temperature [ $^{\circ}\text{C}$ ];

$\alpha_{si}, \alpha_{se}$  – the internal and external thermal surface coefficients [ $(\text{m}^2 \cdot \text{K})/\text{W}$ ].

In the considered case, the  $Q_L$  value determines the computational heat loss for a 1 m high cylindrical wall section (partition with a cylindrical cross-section). In contrast, the  $Q_{\text{sph}}$  value defines the heat loss through the partition, involving the whole surface of the sphere. However, this formula can be adapted to typical shapes used in construction, e.g. half sphere or spherical bowl. In the formulas cited in Table 1, the diameter of the cylinder or sphere is used – while in the rest of this article, the curvature is described by the radius of curvature  $R = D/2$ , using appropriate indexes (Fig. 2).

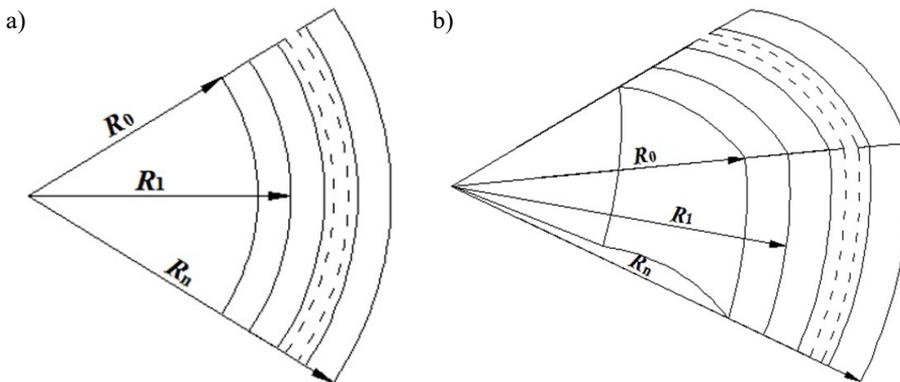


Fig. 2. Scheme designation of the radius of curvature of the layered partition:  
a) cylindrical, b) spherical (*author's study*)

## 2. Assumptions for calculation examples

The analysis was carried out on two examples of partitions: a partition with a cylinder cross-section (cylindrical partition) and a dome (spherical partition). In both cases, heat loss was determined on the basis of exact formulas (from Table 1) and in an approximate way using the  $U_p$  value, as for a flat partition. The heat transfer surface  $F$  has been determined in three variants:

- for the internal radius  $R_0$ ;
- for the external radius  $R_n$ ;
- for mean radius  $R_m$  (mean value from  $R_0$  and  $R_n$ ).

### Example 1

The cylindrical partition,  $n = 3$ , the following arrangement of layers was adopted:

- ceramic block:  $\lambda_1 = 0.4 \text{ W}/(\text{m} \cdot \text{K})$ ,  $d_1 = 0.25 \text{ m}$ ;
- thermal insulation:  $\lambda_2 = 0.04 \text{ W}/(\text{m} \cdot \text{K})$ ,  $d_2 = 0.18 \text{ m}$ ;
- ceramic brick:  $\lambda_3 = 0.77 \text{ W}/(\text{m} \cdot \text{K})$ ,  $d_3 = 0.12 \text{ m}$ .

To simplify the analysis, the thin finishing layers with low thermal resistance were omitted. Thermal surface coefficients were adopted as for the horizontal heat transfer through the partition (PN-EN ISO 6946:2017-10):

- $\alpha_{si} = 7.692 \text{ (m}^2 \cdot \text{K)/W}$ ;
- $\alpha_{se} = 25 \text{ (m}^2 \cdot \text{K)/W}$ .

Calculated values of internal and external air temperature were adopted:

- $t_i = 20^\circ\text{C}$ ;
- $t_e = -20^\circ\text{C}$ .

### Example 2

The spherical partition,  $n = 2$ . The calculations were made for one half of the sphere, the following arrangement of layers was adopted:

- concrete shell:  $\lambda_1 = 0.8 \text{ W/(m} \cdot \text{K)}$ ,  $d_1 = 0.12 \text{ m}$ ;
- thermal insulation:  $\lambda_2 = 0.04 \text{ W/(m} \cdot \text{K)}$ ,  $d_2 = 0.20 \text{ m}$ ;

Resistances of heat transfer were assumed as for the heat transfer up through the partition:

- $\alpha_{si} = 10 \text{ (m}^2 \cdot \text{K)/W}$ ;
- $\alpha_{se} = 25 \text{ (m}^2 \cdot \text{K)/W}$ .

Other parameters were adopted as in an example 1.

## 3. Analysis of the results

The results of the heat loss calculations for example 1 are shown in Table 2. The  $Q_L$  value describes the heat loss estimated using exact formulas. The values  $Q_0$ ,  $Q_m$ ,  $Q_3$  describe the estimated heat loss calculated as for a flat partition in three variants, respectively: for internal, mean and external radius. Graphs illustrating the percentage difference between the analysed values in relation to the exact value are presented in Figure 3. A positive value means an overestimation of heat loss, while a negative value means an underestimation of heat loss.

An analogous analysis for example 2 is presented in Table 3 and in Figure 4.

**Table 2.** Heat loss through a three-layer cylindrical partition – description in the text  
(*author's study*)

Internal radius of the partition $R_0$ [m]	Heat loss $Q_L$ [W/m]	Heat loss $Q_0$ [W/m]	Heat loss $Q_m$ [W/m]	Heat loss $Q_3$ [W/m]
2	106.4	92.2	104.9	117.6
5	244.9	230.5	243.2	255.9
10	475.5	461.1	473.8	486.4
15	706.0	691.6	704.3	717.0
0	936.6	922.2	934.8	947.5
30	1397.7	1383.2	1395.9	1408.6

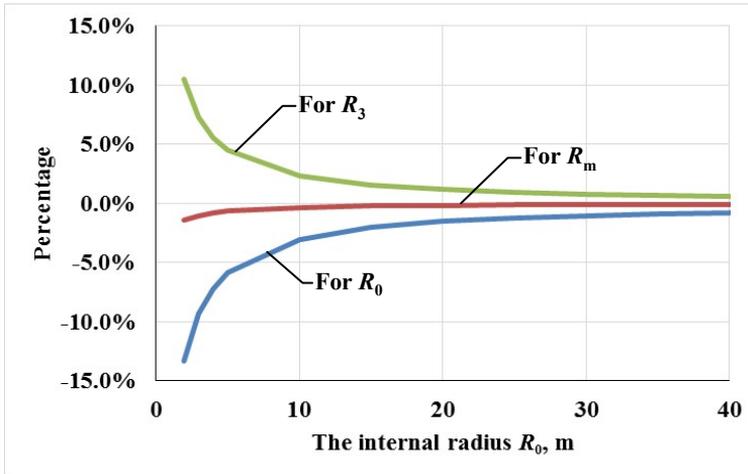


Fig. 3. The percentage inaccuracy of heat loss values calculated as for a flat partition – description in the text (*author’s study*)

Table 3. Heat loss through a double-layer spherical partition – description in the text (*author’s study*)

Internal radius of the partition $R_0$ [m]	Heat loss $Q_{sph}$ [W/m]	Heat loss $Q_0$ [W]	Heat loss $Q_m$ [W]	Heat loss $Q_2$ [W]
2	231.8	190.0	221.7	255.7
5	1289.9	1187.8	1265.0	1344.6
10	4953.9	4751.0	4904.2	5059.9
15	10 993.4	10 689.7	10 919.0	11 150.7
20	19 408.4	19 004.0	19 309.2	19 617.0
30	43 364.9	42 758.9	43 216.2	43 676.0

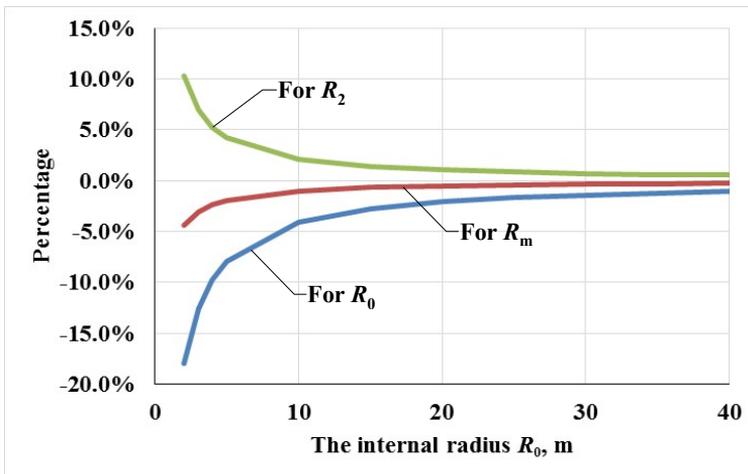


Fig. 4. Percentage inaccuracy of heat loss values calculated as for a flat partition – description in the text (*author’s study*)

Based on the conducted analysis, it has been found that in the case of simplified calculations, consisting of bringing a cylindrical or spherical partition to a flat partition, in most cases the mean radius of curvature can be used to determine the heat transferring surface. In the case of a cylinder, even for a small radius, the inaccuracy does not exceed 1.5%. In the case of a sphere, these inaccuracies exceed 2% for a radius smaller than 5 m.

However, it has been shown that the assumption of the external or internal radius for calculations can lead to an underestimation or overestimation of the computational heat loss. For a large radius, i.e. greater than 10 m for the cylinder and 15 m for the sphere, these inaccuracies do not exceed 3%. For a small radius, these values can be significant, e.g. after adopting an internal radius of 2 m instead of a mean radius, an underestimation of heat loss for the analysed exemplary partitions is: 13.3% for the cylinder and 18% for the sphere.

## Conclusions

The article shows that the value of computational heat loss through curvilinear partitions, estimated by an analogy to flat partitions using the classic  $U$  value, differs from the exact value. The value of this inaccuracy depends on which radius of curvature is used for the calculation of the surface area: internal, external or mean for the partition. In the analysed examples of cylindrical and spherical partitions, adoption of the mean radius led to inaccuracies exceeding 2%, for radiuses of curvature less than 5 m. In the case of assuming the internal or external radius for the calculation, these inaccuracies strongly increase at radiuses smaller than 10 m and for the internal radius of 2 m reached 13.3% for the cylinder and 18% for the sphere.

Therefore, it should be stated that in the case of template practical calculations performed using computer software, the basis for calculating the field of curvilinear heat transferring surfaces should be the mean radius of the curvature of the layered partition, which reduces the risk of calculation inaccuracies. It can be assumed that similar relationships occur for surfaces with an irregular radius of curvature (elliptic, paraboloidal shapes). In this case, the mean value from the external and internal surface area of the solid is proposed as a definitive one.

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